Stochastic analysis of transient flow to a well in a heterogeneous aquifer

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Introduction:

The quantitative analysis of subsurface fluid flow is essential in managing and protecting the largest source of potable water on Earth. The term groundwater is usually used to describe water that is beneath the water table within geologic formations that are fully saturated. Saturated geologic units, which can store and transmit considerable quantities of water under natural physical forces, are called aquifers. There are two main types of aquifers, confined and unconfined. An unconfined aquifer is one where the upper boundary is the water table and the lower boundary is in effect impermeable. A confined aquifer is one that is bound above and below by essentially impermeable layers of geologic units [1]. The study of confined aquifers will be of interest to this research.

The geology of aquifers is usually very complex and displays both systematic and random spatial variations in hydraulic properties on varying scales. Thus, the forces that govern groundwater flow are generally uncertain in their exact nature, magnitude, and spatio-temporal distribution. Traditional deterministic methods of analyses cannot account for this uncertainty and are therefore not sufficient to accurately model realistic aquifer characteristics. In recent years, the response has been a shift to utilize geostatistics and model the subsurface using stochastic analyses.

The areas of interest to this study are groundwater flow velocity and direction as well as aquifer storage capacity. Theses are in turn dependent on the properties of the aquifer, most importantly hydraulic conductivity and storativity. Hydraulic conductivity is a measure of the ability of an aquifer to transmit water. Storativity is a measure of the amount of storage in an aquifer with known thickness. The most common method used to determine aquifer properties is pump test analysis of transient flow. The so-called Theis equation, most commonly used to quantify the data from the pump tests, is valid for an isotropic, homogeneous medium with an infinite radial extent that has a fully penetrating well which starts with an initial zero drawdown. It is the intention of this research to develop a methodology to infer medium properties from pressure interference tests for heterogeneous aquifers while taking into account the uncertain nature of the subsurface. To do this, both analytical and numerical methods are developed and employed. Dr. Daniel Tartakovsky developed an analytical stochastic solution based on recursive approximations of exact space-time moment equations. It is expected that the range of its applicability to especially heterogeneous aquifers will be limited due to its low-order of approximation [2]. To test the validity of this analytical solution as well as provide additional information about the pump test analysis outside the domain of the analytical solution, numerical Monte Carlo simulations are being utilized.

Problem Formulation:

In a confined aquifer, there is a fully penetrating well of zero radius discharging at a constant deterministic rate, Q. A dirichlet boundary, deterministic H_0, is prescribed on an outer circular boundary and located an infinite distance from the pumping well location. With the origin of a polar coordinate system (r,θ) at the center of the well, the problem is defined mathematically using the vertically averaged flow equation as:

$$S\frac{\partial h}{\partial t} = \nabla\bullet(T\nabla h)-Q\delta(\mathrm{x})$$

Subject to the initial and boundary conditions:

$$h(\mathbf{r},t = 0) = H_0 \qquad \& \qquad h(|\mathbf{r}|\to\infty,t) = H_0$$

The variables are S for storativity, h for hydraulic head, ∇ for the gradient operator in polar coordinates, T for a random scalar transmissivity field, and δ for the Dirac delta function. The natural logarithm, Y(x) = lnT(x), of transmissivity forms a spatially correlated, statistically homogeneous random function of the cylindrical space coordinate $\mathbf{r} = (\rho,\theta)$ and is assumed to be normal with mean <Y>, where $\mathbf{r}$ is radial distance from the well and θ is horizontal angle measured relative to an arbitrary radius.

To produce the numerical Monte Carlo simulations, a finite element code written by Dr. Ming Ye was used [3]. It employs an iterative method with an option to use different pre-conditioners. The iterative method is a quasi-minimal residual (QMR) Algorithm, which is a type of conjugate gradient method in Krylov-subspace used for large non-Hermitian linear systems [4]. The domain is square and contains quadrilateral elements with sides dx=dy=0.2, with a pumping well located in the middle of the grid. An analytical solution for a homogeneous aquifer with the properties listed above was used to parameterize the iterative code. To do this, different pre-conditioners and tolerances were modified and then checked against the analytical solution.

Homogeneous Aquifer Problem:

The solution used to check the numerical method was the Theis equation derived from a continuous line source with vertical averaging to apply to two-dimensions. The methods for comparison were type curve-matching, contour plots of hydraulic head and relative error, and column cross-sections of hydraulic head through time. There were two main objectives for this homogeneous analysis. The first was to define the point in time when the analytical solution and numerical solution diverge, called t*. The second was to determine a domain size that was large enough that boundary effects were restricted to the edges and did not affect the interior. This was because the analytical solution is for an infinite domain and numerical methods require a finite domain. These two pieces of information are critical for comparing the validity of the analytical moment equations against the Monte Carlo simulations as now the time and space for which the numerical solution is valid is known.

After completing a large number of experiments with the different pre-conditioners and tolerances, there were two versions of the iterative code that best matched the analytical solution. Both were variations of the Incomplete LU factorization technique called Dual Threshold Incomplete LU factorization (ILUT). The first version was a Left ILUT pre-conditioner and the second version was a Two-sided ILUT pre-conditioner. Both showed approximately the same accuracy when compared to the analytical solution, in terms of relative error. After multiple domain sizes were tested, the conclusion was that the best grid size was 250x250 (63001 nodes). Using a time step from 1 to 20, this grid size gave a t* = 14 and the boundary effects were no longer present after node 7 on all sides, giving a useable grid size of 243x243 (59049 nodes).

Heterogeneous Aquifer Problem:

The statistically homogeneous random log transmissivity, Y(x), values were generated using the program SGSIM, which allows the user to choose the covariance function, variance value, nugget effect, and correlation length. Though one of the distinguishing features of the analytical solution is that it can utilize any covariance function defined at the start, Dr. Tartakovsky's comparative solution used a Gaussian covariance. Thus all Y(x) values were generated using a statistically isotropic Gaussian spatial covariance function. Various correlation lengths and variances will be used in later research but initial experiments used variance = 1 and correlation length =1. A peculiar feature of SGSIM is that when using a Gaussian covariance function, the nugget has to be non-zero or the results will be unstable. This poses a problem, as the analytical solution uses a zero nugget effect. To resolve this issue, realizations were generated with multiple nugget effects. These realizations were then run in the Monte Carlo simulations and the results compared. The conclusion was that there was no difference in head values at multiple locations throughout the grid across time between a nugget of 1×10^{-9} and 1×10^{-10} and that stability in the head values was preserved. Thus the nugget of 1×10^{-10} was chosen as the best value to simulate a zero nugget effect in the comparison. Currently, Monte Carlo simulations of 2000 and 4000 realizations are being run on the 250x250 grid and will be done for various variances and correlation lengths.

Future Research:

Once the validity of the analytical solution has been thoroughly tested, methods will be developed to use this solution in conjunction with a numerical method to infer the spatial statistics of medium permeability and storativity from pressure inference tests either by inversion or graphical methods. This novel approach will then be tested on synthetically generated data. Finally, the method will be applied to existing data from a University of Arizona experimental site consisting of fractured tuffs near Superior, Arizona.

References:

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